

UDC 621.7.04:531.38:004.94

DOI: 10.20535/2077-7264.4(86).2024.334868

© P. Kyrychok, Doctor of Technical Sciences, Professor,
D. Paliukh, Postgraduate Student, Igor Sikorsky
Kyiv Polytechnic Institute, Kyiv, Ukraine

MECHANICO-MATHEMATICAL MODELING OF THE PROFILING PROCESS OF METAL STRIPS FOR FOLDING MECHANISMS

The object of this study is the process of profile formation in folding strips through deformation of metal plates using mechanical and mathematical modeling. The aim is to determine the influence of the physical and mechanical properties of materials and the type of profile on stress distribution and geometry formation during bending.

The results can be implemented in the engineering design of profiled strips with predictable geometric parameters, improved stability, and wear resistance, tailored to specific operating conditions.

Keywords: profile formation; folding strips; finite element method; deformation modeling; physical and mechanical properties.

Introduction

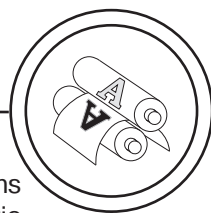
The forming processes of metal strips with a predefined profile geometry are widely used in the manufacturing of complex technical components operating under bending, guided displacement, or precise positioning conditions. The importance of such processes lies in the need to ensure stable geometry through a single-step deformation without subsequent heat treatment or straightening operations.

For example, the production of integral covers for book and magazine products includes folding and flap-gluing operations, where the profile-folding strips play a cri-

tical role. These operations are performed on high-throughput folding-gluing lines, and the strips are responsible for ensuring accurate profile formation and geometric stability under multiple deformation cycles.

During high-speed sheet passage through the folding mechanism, these strips are subjected to intensive loading, including cyclic friction and localized plastic deformation. This leads to surface wear and geometric instability, which critically affects the functional reliability of folding systems.

A review of scientific literature shows a large number of studies related to sheet metal bending,



cold rolling, leveling, and straightening. These works mainly focus on analytical or numerical models for stress fields and residual strain calculations. However, most of them do not thoroughly consider the physical and mechanical properties of specific structural materials or their effect on profile accuracy, especially under single loading scenarios.

One of the key unresolved issues remains the development of a unified mechanical-mathematical model capable of describing the elastoplastic deformation behavior of profiled strips, taking into account tool loading types, indenter geometry, material properties, springback effects, and residual stress development.

The final shape prediction after deformation is complex, as even with the same profile geometry, different metals (e.g., steel, titanium, aluminum, brass) demonstrate significant differences in flow characteristics, stress localization, strain patterns, and springback effects.

In modern engineering design, there is a growing need for models that allow for profile parameter prediction without extensive physical testing. Despite advances in plastic deformation modeling, comparative analysis of dissimilar materials in the context of profile formation remains insufficiently explored.

In particular, there is limited data on the deformation behavior of widely used engineering materials such as cold-rolled low-carbon steel DC01 (AISI 1008), stainless steel AISI 321, commercially pure titanium Grade 2, aluminum alloy Al 6061, and brass L63. These

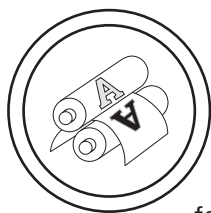
materials differ significantly in terms of plasticity, stiffness, and elastic recovery, which precludes a one-size-fits-all modeling approach.

The lack of a universal methodology for profile formation assessment across different metals complicates engineering optimization, stress-strain state prediction, and the design of efficient folding mechanisms.

Therefore, the scientific novelty and relevance of this study lie in the development of a universal approach for modeling the elastoplastic behavior of strips made of different materials, considering real technological and geometric parameters. This will enable accurate prediction of profile forming results and improve the performance precision of folding mechanisms.

In contemporary scientific literature, considerable attention is paid to the mathematical and numerical modeling of metal strip processing, particularly profiling, bending, leveling, and cold rolling. In most cases, research focuses on residual stress distribution, the influence of equipment or material parameters on the stress-strain state, and the modeling of deformations using the finite element method (FEM).

Article [1] reviews recent advances in tribology, including mechanisms of friction and wear during contact in deformation zones. While a wide range of examples is provided, the study does not focus on the bending of metal strips. Thus, the tribological findings are useful for modeling contact interactions but do not address profile geometric accuracy.



The materials presented in [2] focus on the development of a method for evaluating residual stresses in the leveling process of metal strips using roller systems. The model accounts for plastic deformation and its influence on the internal stress state. However, the study lacks an analysis of geometric profile stability and the effect of material type on deformation results.

Publication [3] presents a numerical-analytical study of forces in three-roll bending of metal sheets. The model considers contact interactions and force distributions, which are relevant to profiling. Nonetheless, the study does not analyze the geometric accuracy of the profile or compare different materials.

Study [4] investigates the mechanical behavior of RHS steel profiles reinforced with GFRP composites using both experimental and numerical analysis. While the reinforcement's effect on load-bearing capacity is analyzed, the study does not include detailed modeling of the profiling process or geometric transformations specific to folding mechanisms.

Results presented in [5] pertain to the unbending of profiled metal sheets in remanufacturing operations. Particular attention is given to the effects of residual stresses and deformations on geometry recovery. However, the work lacks mathematical or mechanical-geometrical modeling of the initial profiling stage and does not analyze stress distribution in the bending zone when forming complex structural elements.

The study [6] proposes a mathematical model for cold-rolled

strip distortion, considering its initial geometry. The model enables prediction of wave height during rolling. However, it does not provide data on the behavior of different material types or the impact of profile formation in bending elements.

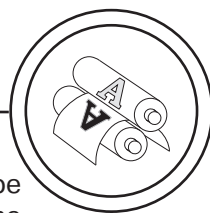
In article [7], a new model for metal strip leveling is developed based on the interaction of 'roller-strip' elements. The model is implemented as a numerical algorithm that accounts for contact forces and plasticity. However, it does not consider the formation of complex profile geometry or multi-step loading scenarios.

Study [8] presents experimental results of strip straightening on multi-roll machines. The work analyzes the technological parameters influencing strip stability. However, it lacks a numerical description of the profiling process and does not account for material variability.

Experimental study [9] addresses the use of FEM for modeling asymmetric rolling processes. Specifically, deformation scenarios under varying roller speeds and center-of-mass shifts are analyzed. Nevertheless, the study does not include examples of bending or formation of geometrically complex profiles.

Publication [10] investigates the effect of cross-section shape on the final form of a bent strip in compression-assisted bending. The study confirms the significance of initial geometry for bending accuracy. However, the model does not account for residual stress distribution or material type.

Source [11] models a multi-roll cold rolling process with an em-



phasis on profile convexity transfer between deformation stages. A 3D FE model is used for analyzing shape formation during multi-stage rolling. However, the model does not address bending as a separate stage or profile forming with springback.

Scientific work [12] proposes a method for developing a theoretical model of bending forces in cold asymmetric rolling. The model estimates stress distribution resulting from asymmetric roller interaction. However, the study does not consider the post-deformation profile state or geometric accuracy.

The research review [13] presents an approach to validating residual stress calculations in structural steels using the contour method. The study combines FEM and experimental investigations on a benchmark sample. However, it lacks analysis of geometric changes or modeling of the profile-forming process during bending.

To generalize the approaches presented in the analyzed sources, a classification of the studies

was conducted based on the type of modeling used. Specifically, the share of studies based on analytical descriptions, numerical simulations (primarily FEM), experimental methods, or their combination as hybrid techniques was analyzed. The results are shown in fig. 1, which illustrates the distribution of studies by predominant model type.

Thus, the available works significantly expand the understanding of individual stages of metal strip processing — bending, leveling, and rolling. However, none of them offers a comprehensive mechanical and mathematical modeling of the strip profiling process for the production of folding mechanism components that simultaneously:

- considers material-dependent parameters (steel, titanium, aluminum, brass),
- describes the distribution of residual stresses,
- predicts elastic springback after loading,

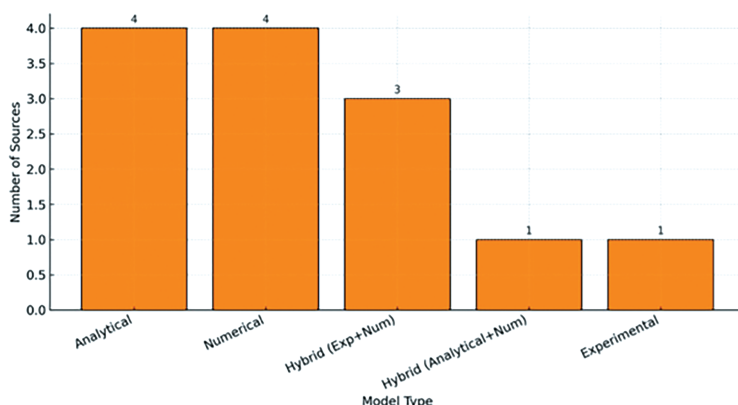
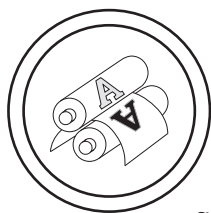


Fig. 1. Distribution of scientific sources by model type (analytical, numerical, experimental, hybrid) in the literature review



— allows assessment of the geometric accuracy of the formed profile.

This highlights the scientific and applied relevance of further research aimed at developing a generalized mechanical and mathematical model that integrates analytical relationships and numerical simulation methods. Such a model should enable the predictive formation of strip profiles, accounting for material-dependent parameters, residual deformations, and industrial application conditions.

Purpose of the study — to improve the accuracy of the profile-forming process of metal strips by developing a mechanical and mathematical model of plastic deformation for a selected group of structural materials, taking into account their physical and mechanical properties and the characteristics of the stress-strain state.

To achieve this purpose, the following tasks must be solved:

— To perform a comparative analysis of the physical and mechanical properties of a selected group of structural materials (AISI 1008, AISI 321, Ti Grade 2, Al 6061, L63) in terms of their suitability for profile plastic deformation, and to evaluate the influence of these properties on profile-forming parameters.

— To develop a geometric and mechanical-mathematical model of the profiling process of metal strips that describes the stress-strain state in the strip cross-section, taking into account the elastic-plastic behavior of the material and its contact interaction with the tooling.

—To conduct numerical simulation of the profile-forming process using the finite element method (FEM) for each selected material; to identify characteristic deformation zones, stress fields, and the amount of springback.

— To evaluate the influence of material properties on the accuracy of profile formation, to establish deformation patterns, and to provide engineering recommendations for material selection and process conditions aimed at ensuring the stability of the profile geometry after deformation.

Methods

The study utilizes a selection of structural materials differing in mechanical and physical properties to model the process of single plastic deformation aimed at forming a stable profile in metal strips. The selected materials (table 1) include:

— Steel DC01 (1.0330, EN 10130, AISI 1008) — cold-rolled low-carbon steel characterized by high formability and technological adaptability;

— Stainless steel AISI 321 — a chromium-nickel austenitic steel alloyed with titanium, resistant to intergranular corrosion and deformation in high-temperature environments;

— Titanium Grade 2 (UNS R50400) — commercially pure titanium with high corrosion resistance and low density;

— Aluminum alloy 6061 (AlMg1SiCu) — typically used where a combination of low weight, high strength, and good machinability is required;

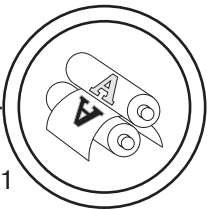


Table 1

Properties of structural materials used in the study

Material	Yield Strength, MPa	Young's Modulus, GPa	Poisson's Ratio	Density, g/cm ³	Strain Hardening Coefficient, n
AISI 1008 Steel	140	210	0.29	7.85	0.20
AISI 321 Stainless Steel	205	190	0.30	7.90	0.25
Titanium Grade 2	275	105	0.34	4.51	0.18
Aluminum 6061	276	69	0.33	2.70	0.16
Brass L63	210	100	0.31	8.50	0.22

— Brass L63 (CuZn37) — a two-phase copper-zinc alloy featuring high plasticity and stable behavior during cold deformation.

Table 1 presents the key physical and mechanical properties of the selected structural materials used for the numerical modeling of the profile forming process. To visually demonstrate the variation of these properties among the chosen materials, comparative diagrams were constructed (fig. 2).

The indicators presented in the diagrams allow for analyzing the differences in the physical and mechanical properties of structural materials that influence the behavior of metal strips during the profile formation process. This, in turn, makes it possible to establish correlations between the structural characteristics of the materials and their ability to undergo plastic deformation under external loading.

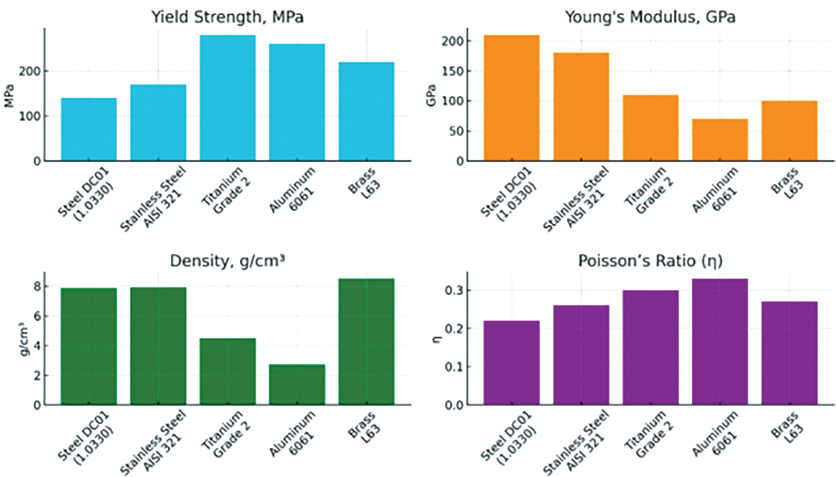
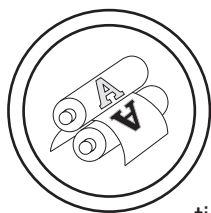


Fig. 2. Comparison of mechanical and physical properties of structural materials used in the modeling of profile formation processes



In this study, the profile formation process of folding strips was numerically modeled using the finite element method (FEM). The purpose of the modeling was to construct the geometric surfaces of the profiled strips and to analyze the influence of material mechanical properties on their behavior under different bending shapes.

The geometric model was represented as rectangular strips with dimensions of $100 \times 20 \times 1$ mm. A regular finite element mesh with a 5 mm step size was used in the calculations. The element type was a 3D solid. The simulation was performed under a static nonlinear formulation, taking into account large deformations. The deformation force was applied in the form of a controlled displacement that transformed the initial flat strip into one of four target profile shapes: logistic, sinusoidal, parabolic, and involute. The profile axis was oriented along the length of the strip. The material model was implemented as an elastic-plastic relationship using the von Mises yield criterion [14].

For the simulation, a conditionally rigid scheme of the profiling tool was used, in which the contact between the tool surface and the strip was modeled using nonlinear contact constraints, incorporating Coulomb friction (coefficient of friction — 0.12) (fig. 2).

The material properties of each material were defined using an elastic-plastic model with isotropic hardening, accounting for the nonlinear stress range (up to 20 % strain).

The analysis of results included the construction of the deformed geometry for each profile type, eval-

uation of stress distribution, identification of plastic deformation zones, and determination of the influence of mechanical properties on the accuracy of profile formation.

The modeling results were compared for each of the five materials to identify the patterns in how physical and mechanical characteristics affect profiling quality, particularly geometric reproduction accuracy and the stress-strain state in the deformation zone. The results are presented as 3D visualizations, stress-strain diagrams, and comparative tables.

Results

A simplified geometric model of the plastic profile deformation process of a metal strip is shown in the diagram (fig. 3). The initial strip of rectangular cross-section is subjected to local loading by means of a profiled indenter, whose shape determines the final geometry of the deformed zone.

The model accounts for contact interactions between the indenter and the metal strip, as well as nonlinear effects occurring in the plastic deformation zone. Based on this geometric model, boundary conditions were formulated for further numerical analysis of the profile plastic deformation process. In particular, the influence of the elastoplastic properties of materials on the stability of the formed geometry and the uniformity of strain distribution in the contact zone with the indenter was investigated. For preliminary assessment of shape stability, a comparative analysis of materials based on their mechanical prop-

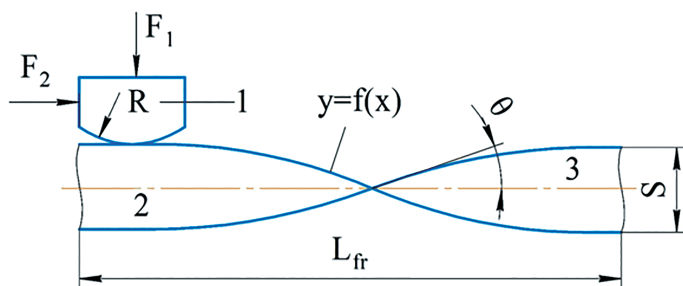
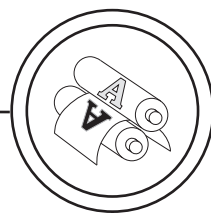


Fig. 3. Simplified geometric model of the profile plastic deformation process of a metal strip under the action of an indenter: 1 — spherical indenter; 2 — metal strip before deformation; 3 — profiled zone after plastic deformation; F_1 , F_2 — deformation forces; R — radius of the spherical forming surface of the indenter; L_{fr} — deformation zone length; S — width of the final deformed section; θ — profile deformation angle; $y = f(x)$ — function describing the shape of the profile

erties was performed. The corresponding results are presented in fig. 4.

The diagram presents normalized formability scores for five structural materials, obtained through an analytical calculation of the product of Young's modulus and yield strength (as indicators of stiffness and resistance to plastic deformation). In this context, formability reflects the material's ability to retain the imparted shape after

load removal without elastic recovery or geometric instability of the profile. The obtained values were normalized within the $[0; 1]$ interval for ease of comparison:

- AISI 321 — 1.00 (maximum stiffness and high yield strength);
- AISI 1008 — 0.88 (high stiffness with moderate yield strength);
- L63 — 0.55 (high plasticity with moderate stiffness);
- Ti Grade 2 — 0.52 (high yield strength with low stiffness);

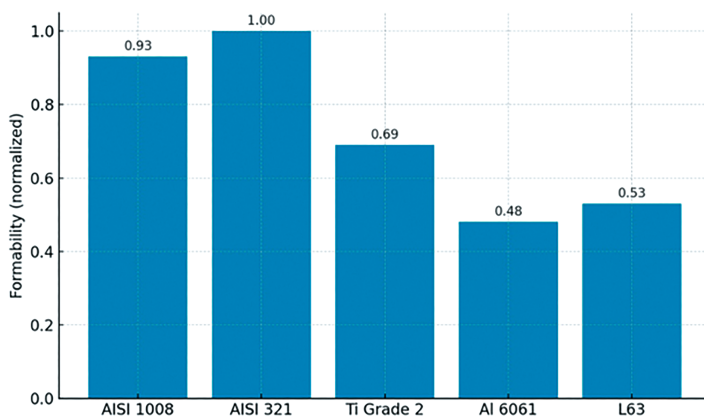
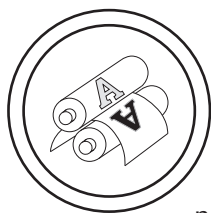


Fig. 4. Formability evaluation of the investigated materials based on reference mechanical properties



— Al 6061 — 0.42 (lowest stiffness and relatively high plasticity).

This approach enables a quantitative comparison of the materials' potential to maintain shape following plastic deformation during profiling.

Given the obtained formability assessments, it is necessary to select folding plate geometries that minimize the influence of elastic springback and support stable shape formation regardless of the material type. In this context, it is advisable to compare analytical models of profiles that differ in curvature characteristics and potential to ensure uniform bending.

Figure 5 presents three basic profile types — sinusoidal, parabolic, and logistic — that may be used as functional geometries of folding plates in the technological process of deforming flat metal strips with varying stiffness and plasticity indices.

Among the smooth geometric curves (fig. 5) that can be used to describe the deformation trajectory of folding plates, the sinusoidal function is analyzed first. In

particular, a half-period of the sine wave enables the formation of a symmetric bend with a gradual entry and exit, which is suitable for modeling repeated contact deformations. The sinusoidal profile is defined by the expression:

$$y(x) = x \cdot \sin\left(\frac{\pi x}{2L}\right), \quad (1)$$

where $y(x)$ — profile height at point x ; h — amplitude of the bend (maximum height); L — length of the profiled segment.

The shape of the curve provides a smooth rise and fall of the profile height, ensuring minimal gradient loading during plate profiling. Due to this, the sinusoidal profile is suitable for materials that are highly sensitive to local stress concentration, particularly aluminum and titanium alloys.

Advantages of the sinusoidal profile include:

- Moderate conformity to the natural bending trajectory under translational loading;
- Absence of abrupt curvature changes;

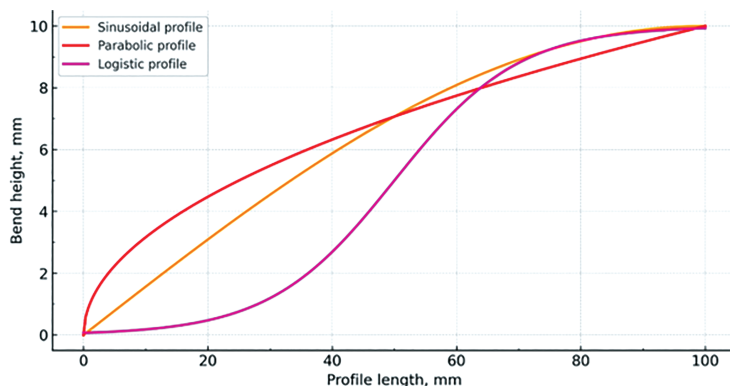
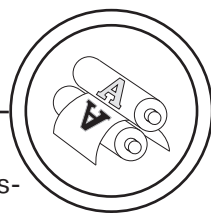


Fig. 5. Geometrical profiles of folding plates constructed based on sinusoidal, parabolic, and logistic functions



— Applicability in finite element modeling environments;

Easy implementation through cold extrusion or template-based bending.

These properties make the sinusoidal profile suitable for applications where stress concentration reduction and uniform load distribution along the length of the profiled plate are critical. This ensures a smooth alignment of cover elements during bending along the folding plate profile, reducing the risk of local deformations and loss of geometric stability of the formed profile.

One of the simplest and most effective methods for mathematically describing a smooth bend for folding plate formation is the use of a parabolic profile (fig. 5). This approach allows modeling of gradual shape formation, in which the bending trajectory changes with acceleration — typical for the plastic deformation of metal plates under external loading.

The parabolic function is expressed as follows:

$$y(x) = h \cdot \left(1 - \left(1 - \frac{x}{L} \right)^2 \right), \quad (2)$$

where $y(x)$ — vertical deflection of the profile at point x ; h — maximum bending height; L — profile length.

This function starts from zero height, gradually increases, and reaches the maximum value h at the end of the segment $x = L$, which ensures a smooth rise in deformation during the profiling process of the strips.

Advantages of the parabolic profile:

— simplicity of analytical description;

— smooth increase in bending without abrupt changes in curvature;

— suitability for numerical analysis and experimental reproduction via roll or stamping deformation.

Due to these characteristics, the parabolic function is widely used for constructing profile trajectories in technical forming tasks.

One of the promising approaches to profile formation is the use of the logistic function (fig. 5), which provides a smooth variation of curvature along the length of the guide. This approach enables the modeling of a gradual transition of the folding plate surface toward the cover unfolding plane without abrupt trajectory changes, thereby reducing local stress concentrations in the contact zone.

The logistic function has the form:

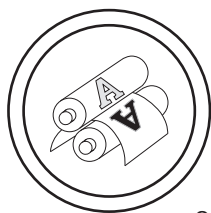
$$y(x) = \frac{h}{1 + e^{-k(x-x_0)}}, \quad (3)$$

where $y(x)$ — vertical deviation of the profile at point x ; h — maximum bending height; x_0 — coordinate of the inflection point (center of the transition zone); k — steepness parameter determining the curvature variation pattern.

Advantages of using the logistic function for geometric profile modeling include:

— symmetry with respect to the axis $x = x_0$;

— controlled rate of approach to the asymptotic height h ;



- absence of discontinuities or abrupt changes in the first derivative, ensuring a smooth bending transition;

- flexible adjustment of the profile to specific material properties via the parameter k .

The shape of the curve is characterized by a smooth S-shaped increase, which ensures controlled material deformation along the profile length. Therefore, the logistic profile serves as a mathematical model that allows for predictable control of the folding plate formation process under a single plastic deformation load.

Following the analysis of three basic profile curves, it is advisable to consider the involute curve as an alternative geometric shape with promising engineering applications in profiling flat metal plates.

The involute curve (involute of a circle) is a classical element of technical geometry that is widely used in mechanical engineering for constructing meshing surfaces. In the context of folding plate formation, such a curve enables controlled and gradual contact bending with uniform strain distribution along the guide line.

The involute is defined by the following parametric equations:

$$\begin{aligned}x(t) &= R \cdot (\cos t + t \cdot \sin t) \\y(t) &= R \cdot (\sin t - t \cdot \cos t),\end{aligned}\quad (4)$$

where R — the radius of the base circle; t — a parameter that determines the length of the curve's unwinding.

As the parameter t varies from 0 to a chosen value, the involute unfolds from the base circle and

forms a curve characterized by a monotonic increase in curvature. This property is important for gradual bending without localized material overloading.

Advantages of the involute profile include:

- uniform curvature progression along the length of the bend,

- minimization of stress concentrations in contact zones,

- high accuracy in predicting the bending trajectory,

- geometric compatibility with profile guides in mechanical systems.

Fig. 6 presents a fragment of the involute curve for $R = 20$ mm, constructed within the parameter range $t = [0, \pi/2]$. This corresponds to the natural bending trajectory of the flaps of integral covers during movement along the guide surface.

Thus, the involute dependency represents a mathematically justified option for constructing profiles that ensure controlled plastic deformation of the material with predictable geometric parameters. The advantage of this geometry lies in its ability to adapt to different loading conditions, enabling gradual deformation growth without the formation of local stress concentrations.

For various structural metals from the sample set (AISI 321, AISI 1008, L63, Ti Grade 2, Al 6061), the suitability of a specific profile depends on the combination of their Young's modulus, yield strength, and elastic springback. Table 2 presents a comparative assessment of the recommended profiles for each metal, taking into account their formability and deformation behavior characteristics.

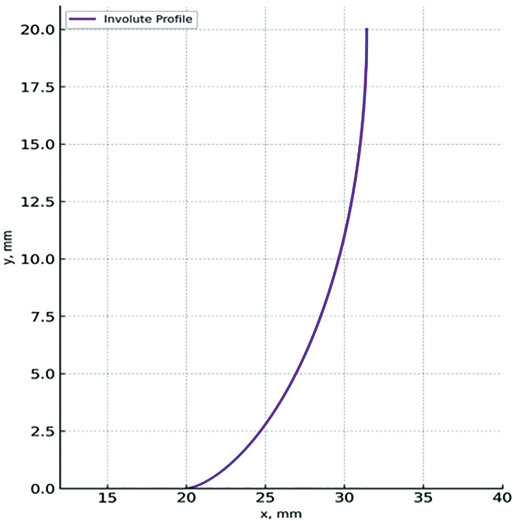
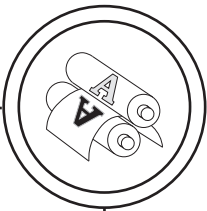


Fig. 6. Involute curve as a geometric basis for constructing a bending profile with uniform strain development

Based on the generalized data presented in table 2, a graphical interpretation was performed to illustrate the relationship between types of geometric profiles and structural materials. This visualization enables a rapid assessment of the suitability of specific profile shapes for use with materials possessing different physical and mechanical properties.

Fig. 7 presents a formability suitability chart that visualizes the correspondence between geometric profile types (logistic, parabolic, sinusoidal, involute) and structural materials (AISI 321, AISI 1008, L63, Ti Grade 2, Al 6061) based on key mechanical characteristics.

This binary comparison provides a justified assessment of the

Table 2
Recommended geometric profiles for structural materials

Material	Recommended profile	Justification
AISI 321	Logistic/Parabolic	Maximum shape stability; suitable for precise bending
AISI 1008	Parabolic/Logistic	Optimal stiffness; deformation without overload
L63	Sinusoidal	Flexible; requires gentle load distribution
Ti Grade 2	Sinusoidal/Involute	Plastic; requires uniform profile formation
Al 6061	Sinusoidal/Parabolic	Prone to instability; needs smooth curvature transitions



feasibility of using a given profile geometry for each material, taking into account their ability to ensure stable shape formation under single plastic loading without loss of geometric stability of the formed profile.

For a more in-depth analysis of the shape formation of profiled strips, it is advisable to consider the problem of continuous bending, which accounts for motion along a predefined geometric trajectory. This approach enables the modeling of real conditions in which a metal strip passes through a profiling tool that forms the desired surface curve.

The problem of two-dimensional bending of a flat metal strip moving along a profiling indenter with a prescribed curve $y = f(x)$ (e.g., sinusoidal, parabolic, logistic, or involute) is considered.

Kinematic assumptions:

- Deformation occurs in the bending plane xOy ;
- Contact between the strip and the tool occurs along a curved surface under constant pressure;

— Strip thickness changes negligibly (small bending assumption).

The material behavior of the strips is described using an elasto-plastic model with isotropic hardening — the Hooke–Hencky model [15], which captures both elastic and plastic stages of deformation. This model allows for an accurate representation of the stress–strain state in the strip cross-section under single loading conditions, incorporating the effects of elastic springback and material hardening. Its application within the framework of mechanical-mathematical modeling provides a technically substantiated engineering assessment of shape formation, deformation zones, and residual stresses in the profiling process.

Model components:

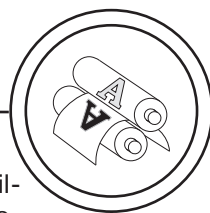
1. Hooke’s Law (linear elastic region):

Describes the linear relationship between stress and strain up to the yield point. The formula for the uniaxial case is:

$$\sigma = E \cdot \varepsilon, \quad (5)$$

Material	AISI 1008	0	1	1	0
	AISI 321	0	1	1	0
	Al 6061	0	0	1	1
	Ti Grade 2	1	0	0	1
	Brass L63	0	0	0	1
		Involute	Logistic	Parabolic	Sinusoidal

Fig. 7. Comparison of mechanical and physical properties of structural materials



where σ — stress; ε — strain; E — Young's modulus.

2. Plastic deformation according to the Hencky model:

Accounts for the nonlinear (post-yield) behavior of the material. After reaching the yield strength, the material continues to deform with strain hardening, expressed as:

$$\sigma = \sigma_y + K \cdot \varepsilon^n, \quad (6)$$

where σ — yield strength; K — hardening coefficient; n — strain hardening exponent.

3. Transition conditions:

The model allows a smooth transition from elastic to plastic behavior:

- Hooke's law is valid up to σ_y ,
- beyond that point, the Hencky model applies.

Assumptions:

- Strains are small ($\varepsilon \ll 1$);
- The profiling tool is considered perfectly rigid;
- Friction between the strip and the profiling tool is modeled using the Coulomb friction law [16].

Figure 8 presents a diagram illustrating the Hooke–Hencky model, which describes material deformation across two main stages.

In the elastic region (left part of the curve), stress is linearly dependent on strain in accordance with Hooke's law. In contrast, in the plastic region (right part of the curve), the model accounts for isotropic hardening, i.e., the gradual increase in deformation resistance due to the accumulation of internal structural changes within the material.

During the profiling of a metal strip, it is essential not only to accurately describe the internal stress–strain state of the material but also to consider the contact interaction between the tool and the workpiece. A key factor in this interaction is the frictional force, which affects both the load distribution and the accuracy of the shape formation.

To describe the contact interaction, the classical Coulomb dry friction model is applied. According to this model, the frictional

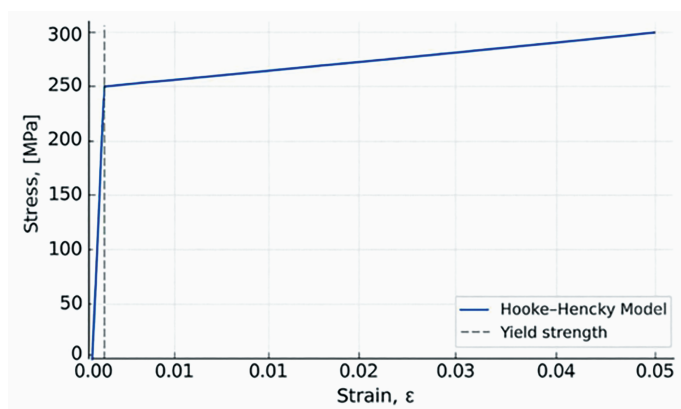
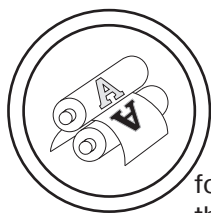


Fig. 8. Stress–strain state of the material according to the Hooke–Hencky model with elastic and plastic deformation regions



force F_t is directly proportional to the normal load N and is defined by the following equation:

$$F_t = \mu \cdot N, \quad (7)$$

where F_t — friction force (N); μ — coefficient of friction (dimensionless), depending on the material pair of the contacting surfaces; N — normal force acting perpendicular to the contact surface (N).

According to Coulomb's law, two friction regimes are considered, which depend on the presence or absence of relative motion between the contacting bodies. These regimes are critical for accurately describing the contact interaction between the profiling tool and the deformed strip during the forming process:

1. Static friction (before motion begins):

$$F_t \leq \mu_s \cdot N, \quad (8)$$

where μ_s — the coefficient of static friction. In this regime, no relative displacement occurs between the surfaces, and the frictional force gradually increases until motion initiates.

2. Kinetic (sliding) friction (during motion):

$$F_t = \mu_k \cdot N, \quad (9)$$

where μ_k — the coefficient of kinetic friction, typically lower than μ_s , and it describes the frictional behavior under sliding contact.

During the interaction between the tool and the metal strip (in rolling or stamping operations), Coulomb's law allows for accounting of frictional forces between the surfaces by:

— defining the resistance to material displacement in the contact zone,

— influencing the distribution of stresses and strains in the plate,

— and being incorporated into the mechanical-mathematical model as boundary conditions on the contact surfaces.

For further analysis, it is advisable to consider the stress-strain state in a characteristic cross-section of the strip, taking into account its physical and mechanical properties defined by the following parameters: E — Young's modulus; μ — Poisson's ratio; σ_y — yield strength; K — hardening coefficient; n — strain hardening exponent.

Based on these fundamental material characteristics, a mathematical model was developed to describe the laws governing elastoplastic deformation of the metallic strip. In this context, it is appropriate to apply constitutive relations that capture both the elastic and plastic behavior of the material during the bending process.

Deformation law:

$$\sigma = K \cdot (\varepsilon_p)^n, \quad (10)$$

where ε_p — the plastic strain.

To evaluate the stress state in the cross-section, the plane bending condition is applied:

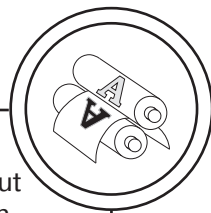
$$\sigma(y) = E \cdot k \cdot y, \quad |y| < y_{el}, \quad (11)$$

where k — the curvature; y_{el} — the boundary of the elastic region.

The plastic zone arises under the condition:

$$\sigma(y) = \sigma_y, \quad |y| > y_{el}. \quad (12)$$

The bending moment considering plastic deformation is determined as:



$$M = \int_{-h/2}^{h/2} \sigma_x(y) \cdot y dy. \quad (13)$$

Contact interaction with the tool:

— A normal contact pressure $p(x)$ is introduced along the contour of the curve $y = f(x)$;

— The friction coefficient μ is accounted for, generating tangential forces:

$$\tau = \mu \cdot p(x). \quad (14)$$

— The contact stresses are included in the boundary conditions of the elastoplastic deformation problem.

Accordingly, for numerical implementation, a variational formulation of the problem is constructed as a minimization of the deformation energy functional:

$$\Pi = \int_{\Omega} \left(\frac{1}{2} \sigma_{ij} \varepsilon_{ij} - \sigma_y \cdot \varepsilon_p \right) d\Omega - \int_{\Gamma} T_i u_i d\Gamma, \quad (15)$$

where Ω — the domain of the strip; Γ — the contact boundary.

The consideration of contact stresses and friction at the tool-workpiece interface enables the formulation of physically justified boundary conditions for constructing a complete mathematical model of the process. Based on the results of this formulation, the key output parameters are determined (table 3), which allow for a quantitative assessment of the forming process, the nature of the stress-strain state, and the elastic springback effects.

The proposed model is suitable for numerical implementation using the finite element method (FEM),

which enables variation of input parameters and analysis of the influence of technological factors, including:

— selection of material (e.g., AISI 321, Ti Grade 2, Al 6061, etc.);

— geometry of the tool (type of curvature);

— magnitude of the applied forming force;

— friction conditions in the tool-material contact zone.

Within the framework of numerical simulation, distributions of von Mises equivalent stresses [14] ($\sigma_{\text{von Mises}}$), equivalent plastic strains, as well as the elastic springback field after unloading were obtained (figs. 9–11). This allows for a comprehensive assessment of the forming behavior and residual stresses in the profile of the metal strip.

In real structural components, stresses occur in multiple directions simultaneously. The von Mises stress provides a means of reducing a complex three-dimensional stress state to a single scalar value, which can be compared to the material's yield strength obtained from a uniaxial tensile test.

The diagram in fig. 9 presents the maximum stress values generated in the materials under tool loading. The highest stress levels are observed in Ti Grade 2 and AISI 321, indicating an intense concentration of internal forces in response to the contact action of the profiling tool.

The diagram (fig. 10) illustrates the pattern of plastic strain accumulation within the material body. The highest level of deformation is observed for the Al 6061 alloy, followed by L63. The lowest plastic

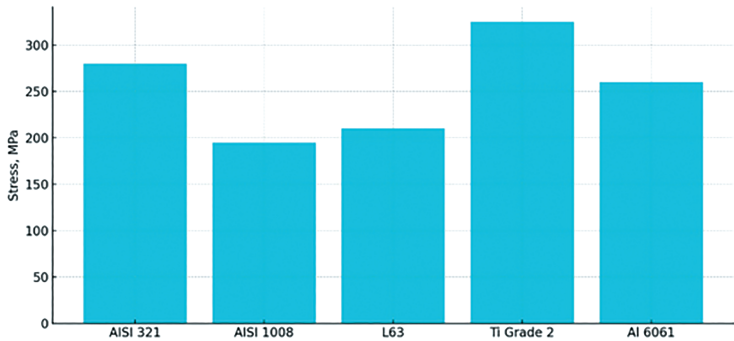
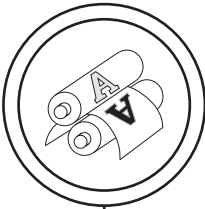


Fig. 9. Comparison of von Mises equivalent stresses for different materials under plastic deformation

Table 3
Key output parameters of the metallic strip deformation model

No	Parameter	Description	Symbol/Unit
1	Distribution of normal stresses	Stresses along the deformation directions (x and y axes) characterizing the stress state	σ_x, σ_y [MPa]
2	Distribution of strains	Relative dimensional changes along x and y directions caused by the applied load	$\varepsilon_x, \varepsilon_y$ [-]
3	Depth of the plastic zone	Thickness of the layer in which irreversible (plastic) deformation occurs	h_{pl} [mm]
4	Curvature of the profile after unloading	Geometric deviation of the profile due to elastic springback	R or $\Delta\kappa$ [1/mm], [mm]
5	Forming forces	Total load required to achieve the specified profile shape	F(form) [N]
6	Contact reactions of the tool	Normal and tangential pressure forces between the tool and the material	$F_{norm}, F_{friction}$ [N]
7	von Mises equivalent stress	Generalized stress measure that determines the onset of plasticity	σ_{eq} or $\sigma_{von Mises}$ [MPa]
8	Equivalent plastic strain	Integrated evaluation of the extent of plastic strain accumulation in the material	$\varepsilon_{pl, eq}$ [-]
9	Field of residual strains	Map of residual strains after unloading, affecting the profile's stability	ε_{res} [-]
10	Displacement vector of nodes	Overall material displacement resulting from the forming process	u_x, u_y [mm]

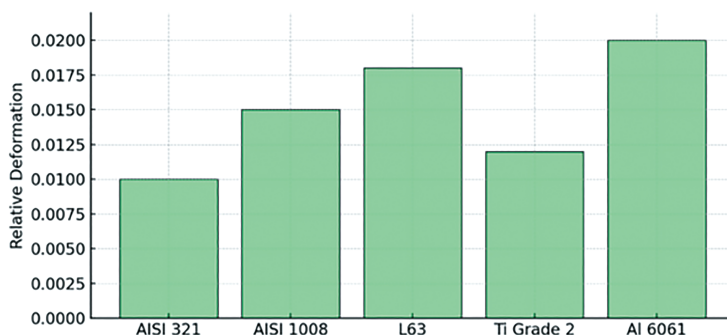
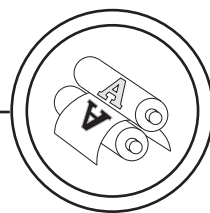


Fig. 10. Comparison of relative plastic strain for various structural materials

strain is exhibited by AISI 321 steel, indicating its limited ability to undergo irreversible deformation under loading. This contrast highlights the varying effectiveness of materials in facilitating plastic forming during bending.

The diagram (fig. 11) presents the magnitude of geometric changes after unloading, which characterizes the elastic recovery effect. This parameter is crucial for evaluating profile accuracy and is determined by the stiffness of the material, particularly its Young's modulus.

Since the degree of elastic springback directly depends on

the distribution of the stress-strain state within the strip, further analysis allows for identifying the nature of spatial interaction between elastic, plastic, and transition zones to ensure the geometric stability of the formed profile.

To identify characteristic deformation zones for each material (AISI 321, AISI 1008, L63, Ti Grade 2, Al 6061), the results of simulation of elastic-plastic profile formation, as well as the distribution of stresses and strains (based on FEM), were used.

In this context, it is appropriate to analyze the geometric characteristics of the shape formation of

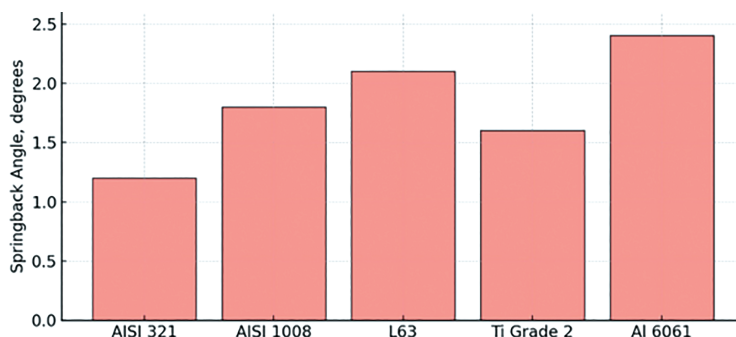
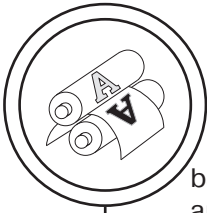


Fig. 11. Springback after unloading for different materials, expressed as the profile recovery angle



basic profile types, which can be applied as functional surfaces of folding plates. Figure 12 presents 3D models of the plates, showing the deflection behavior along the Z-axis depending on the selected profile trajectory. This representation allows for visually assessing the suitability of each function — sinusoidal, logistic, parabolic, and involute — for achieving smooth bending of materials with various physico-mechanical properties.

Based on the obtained three-dimensional models of profile formation in bending elements (fig. 12), a further analysis of the stress–

strain state distribution in the metal was carried out for different materials. This made it possible to generalize the simulation results and construct a representative 3D diagram (Fig. 13) illustrating the typical structure of deformation zones (elastic, transition, and plastic) in the cross-section of a metallic strip.

Such an approach allows for the identification of specific features of material–tool interaction and facilitates prediction of profile behavior under technological loading conditions.

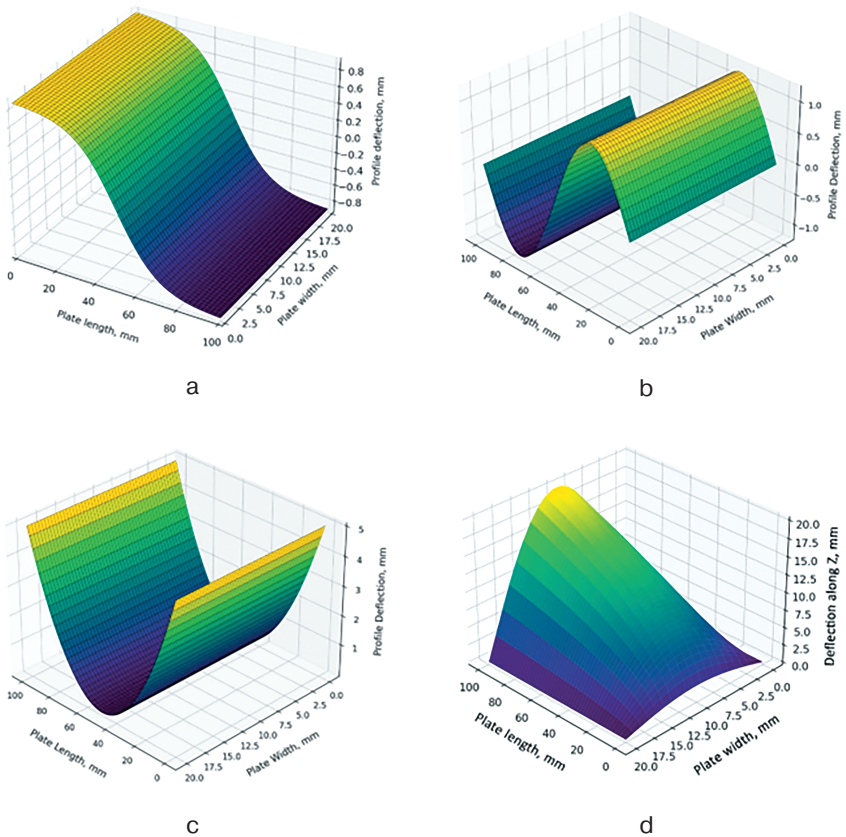
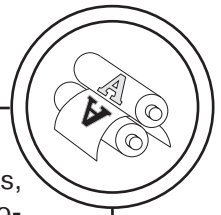


Fig. 12. Geometric models of profile formation in bending elements: a — sinusoidal profile; b — logistic profile; c — parabolic profile; d — involute profile



The 3D diagram visualizes three distinct zones: the region of predominantly elastic deformation, a transition zone with localized plasticity, and a zone of intensive plastic deformation formed near the apex of the bent profile.

The processes of profile formation in metallic strips, as considered in the model, are described within the framework of classical elastoplasticity theory using the differential relations of continuum mechanics. The initial parameters obtained from numerical simulation have analytical interpretations.

The normal stress components in the deformation plane ($\sigma_{x</sub>}$, $\sigma_{y</sub>}$) are determined based on the equilibrium equations for the plane stress problem:

$$\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x &= 0 \\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + f_y &= 0, \end{aligned} \quad (16)$$

where τ_{xy} — the shear stress, and f_x, f_y — the components of body forces.

The corresponding strain components ($\varepsilon_{x</sub>}$, $\varepsilon_{y</sub>}$) are calculated based on the small strain relations:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \end{aligned} \quad (17)$$

where $u(x,y)$ and $v(x,y)$ — the displacements in the x and y directions, respectively.

The generalized assessment of the stress state is performed using the von Mises equivalent stress:

$$\sigma_{eq} = \sqrt{\frac{1}{2} \left[(\sigma_x - \sigma_c)^2 + \sigma_x^2 + \sigma_c^2 + 6\tau_{xc}^2 \right]}, \quad (18)$$

and is compared with the yield strength of the material $\sigma_{y</sub>}$

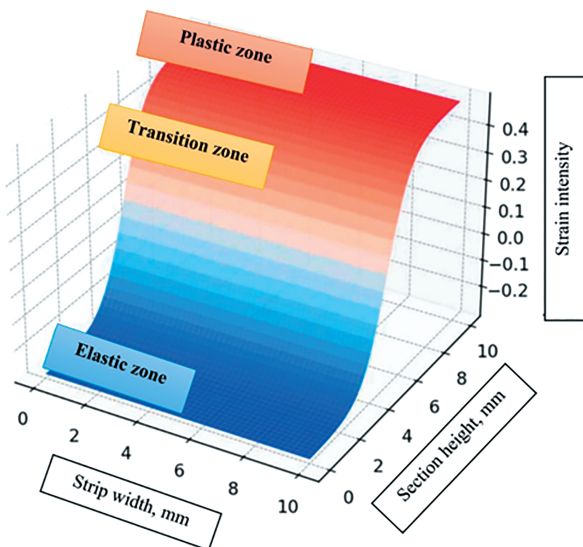
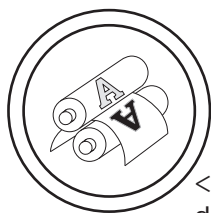


Fig. 13. Generalized diagram of deformation zones in metallic strips



</sub>. The region where this condition is satisfied is considered plastic. The depth of the plastic zone is defined as the maximum coordinate along the thickness where this condition holds true:

$$h_{pl} = \max_y \{y: \sigma_{eq}(x, y) \geq \sigma_y\}. \quad (19)$$

The curvature of the deformed profile after unloading is defined as the second derivative of the normal displacement $w(x)$ with respect to the coordinate x :

$$k_x = \frac{d^2 w}{dx^2}. \quad (20)$$

The springback causes a change in this curvature due to unloading, which is taken into account in the comparative analysis of the curvature before and after the removal of external forces.

The total contact forces acting on the tool are calculated as integrals of the contact pressure and tangential reactions along the contour of the contact area:

$$F_n = \int_{\Gamma} p(x) d\Gamma, \quad F_t = \mu \cdot F_n, \quad (21)$$

where $p(x)$ — the normal contact pressure; μ — the coefficient of friction.

The total force required for profile formation includes both components. To assess the degree of accumulated plastic deformation, the equivalent plastic strain is applied:

$$\varepsilon_{pl,eq} = \sqrt{\frac{2}{3} \varepsilon_{eq}^{pl} \varepsilon_{eq}^{pl}}, \quad (22)$$

where ε_{eq}^{pl} — the components of the plastic strain tensor. After

unloading, part of the deformation remains in the form of residual (irreversible) strains:

$$\varepsilon_{res} = \varepsilon_{tot} - \varepsilon_{el}, \quad (23)$$

and the total displacements of the material are determined as the result of integrating the derivatives of the displacement functions:

$$\vec{u}(x, y) = [u(x, y), v(x, y)]. \quad (24)$$

To numerically solve the problem of elastoplastic deformation with contact interaction, it is necessary to define both the geometry of the computational domain and the appropriate boundary conditions for displacements and loads. To formulate a typical problem setup involving the profile formation of a metallic strip by a rigid tool with a curvilinear profile, the following conditions are adopted:

To ensure positional stability:

$$u(x, 0) = 0, \quad v(x, 0) = 0, \quad (25)$$

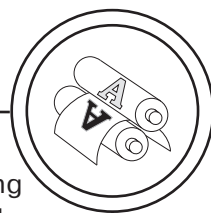
where u, v — the displacement components in the x and y directions, respectively.

These conditions ensure rigid fixation of the lower boundary of the plate. On the axis of symmetry (e.g., $x = 0$), the following boundary conditions are imposed:

$$u(0, y) = 0, \quad \frac{\partial v}{\partial x}(0, y) = 0, \quad (26)$$

which corresponds to the absence of tangential displacements and shifts along the axis of symmetry.

On the contact surface with the rigid tool, a normal pressure $p(x)$ is applied, acting in the direction of the surface normal, and friction is taken into account according to Coulomb's law:



$$\sigma_n = -p(x), \quad \tau_t = \mu \cdot |p(x)|, \quad (27)$$

where σ_n — the normal contact stress; τ_t — the tangential (shear) friction stress, and μ is the coefficient of friction.

On the free boundaries of the plate (e.g., $x = L$ or $y = H$), no external loads are applied, which leads to the following boundary conditions:

$$\sigma_{ij} \cdot n_j = 0, \quad (28)$$

where n_j — the components of the normal vector to the corresponding boundary. This means that no external forces are applied on these boundaries, and the material is free to move along their direction.

Taking into account the specified conditions, the problem is formulated as a variational problem of minimizing the potential deformation energy:

$$\Pi(u, v) = \int_{\Omega} W_{ij} d\Omega - \int_{\Gamma_t} \bar{t}_i \cdot u_i d\Gamma \rightarrow \min, \quad (29)$$

where W — the strain energy density (e.g., based on Hooke's law or a more complex elastoplastic model); $\bar{t}_i$ — represents the surface loads (including contact pressure and friction forces); Γ_t — the portion of the boundary where surface forces act (contact with the tool).

The simulation results make it possible to predict the geometry of the profile after deformation, taking into account the material properties. This enables the selection of an optimal bending shape to achieve the required functionality

and wear resistance of folding strips used in printing and packaging equipment.

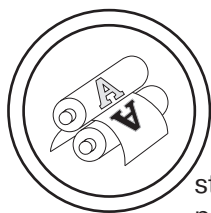
The use of mechanical and mathematical models helps reduce the number of experimental trials required during the design of profiled elements. This reduces material consumption and prototype manufacturing time, which is especially relevant for serial production of profiled metal strips.

The developed models allow for the prediction of springback zones, residual stresses, and potential geometric instability. The obtained results of the profile-forming simulation of metallic strips can be directly applied to the design of mechanisms for folding flaps in book and magazine integral covers.

Discussion

The results obtained in the course of numerical modeling have confirmed the hypothesis about the influence of the shape of the deforming surface on the character and magnitude of the deflection of metal plates used in folding systems. The mirrored involute deformation model demonstrated a more symmetrical and gradual displacement profile compared to classical forms, which may lead to a more uniform stress distribution in the folding area.

A significant feature of the mirrored involute profile is the shift of the maximum deflection zone to the center of the plate's width, which differs from results for plates deformed by linear or parabolic profiles, where deformation maxima are typically observed near the bending edge. This behavior is favorable in terms of reducing local peak



stresses and, consequently, improving the durability of the guiding elements during repeated operation in folding units.

Compared to the results reported in [7, 9], where sinusoidal and parabolic profiles were considered, the involute profile ensures a smoother stress transition along the width of the plate. This difference can be attributed to the geometry of the evolute, which provides continuous variation of curvature along the contact trajectory. The findings also align with the conclusions presented in [11], which emphasized the importance of surface shaping in extending the service life of folding tools.

A limitation of this study is the absence of physical experiments to validate the simulated deformation fields. Future work should include experimental strain mapping using methods such as digital image correlation (DIC) to verify the numerical results and confirm the distribution of deflection across the surface of the profiled plate. Moreover, wear simulation and fatigue analysis should be conducted to assess the long-term performance of folding elements with evolute-based deformation.

The originality of the proposed model lies in the combination of mirrored involute surface geometry and the use of a fine finite element mesh, which allowed for a detailed investigation of the deformation behavior under load. These results open up the possibility of optimizing the design of folding plates and indenters to improve the reliability of machines involved in the production of integral book and magazine covers.

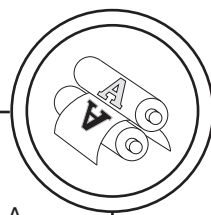
Conclusions

1. A new approach to the mechanical and mathematical modeling of the profiling process for metal strips has been proposed. It combines geometric parameterization of the profile with numerical simulation using the finite element method. This approach enables the reproduction of profile-forming technologies for folding plates, taking into account material properties and loading conditions.

2. The study identified the influence of physical and mechanical properties (elastic modulus, yield strength, and strain hardening coefficient) on the deformation outcomes of profiled strips. It was established that materials with high yield strength (AISI 321, Ti Grade 2) demonstrate higher profile stability, whereas materials with lower stiffness (Al 6061, L63) exhibit greater susceptibility to localized bending.

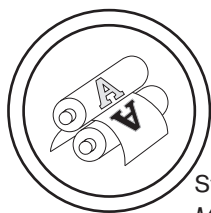
3. Models and visualizations of profile deformation were developed for plates with logistic, sinusoidal, parabolic, and involute profiles. This made it possible to identify the characteristic zones of elastic, plastic, and transitional deformation for each material type, as well as to assess the geometric stability of the resulting shapes.

4. The practical significance of this work lies in the development of a toolset for preliminary evaluation of the geometric efficiency of profiled folding plates made from various materials. The findings of the study can be applied in the design of folding units, elastic components, or profiled parts in the fields of printing technology, mechanical engineering, and materials science.



References

1. Meng, Y., Xu, J., Ma, L., Jin, Z., Prakash, B., Ma, T., & Wang, W. (2022). A review of advances in tribology in 2020–2021. *Friction*, 10(10), 1443–1595. <https://doi.org/10.1007/s40544-022-0685-7>.
2. Zhu, X., Wang, X., & Wu, H. (2024). Calculation method and analysis of residual stress in the strip bending roller straightening process. *Scientific Reports*, 14, 9149. <https://doi.org/10.21203/rs.3.rs-3902940/v1>.
3. Boazu, D., Gavrilescu, I., & Stan, F. (2024). Analytical and Finite Element Analysis of the Rolling Force for the Three-Roller Cylindrical Bending Process. *Materials*, 17(21), 5230. <https://doi.org/10.3390/ma17215230>.
4. Boru E., Aydın E., & Sadid M. S. (2023). Investigation of bending behaviors of GFRP-strengthened steel RHS profiles with experimental and numerical models. *Buildings*, Vol. 13, No. 5, 1216. <https://doi.org/10.3390/buildings13051216>.
5. Farioli D., Kaya E., & Strano M. (2024). Flattening of bent metal sheets as a remanufacturing operation. *Journal of Remanufacturing*, Vol. 14, No. 2, 293–314. <https://doi.org/10.1007/s13243-024-00146-3>.
6. Li, Y., Wen, J., Lin, H., Yu, M., & Wang, F. (2022). Mathematical modelling of online warping height of cold-rolled thin strip steel. *International Journal of Steel Structures*, 22(4), 913–919. <https://doi.org/10.1007/s13296-022-00615-0>.
7. Yi, G., Wang, Z., & Hu, Z. (2020). A Novel Modeling Method in Metal Strip Leveling Based on a RollStrip Unit. *Mathematical Problems in Engineering*, 8, 1–16. <https://doi.org/10.1155/2020/1486864>.
8. Gribkov, E. P., Dobronosov, Y. K., Kovalenko, A. K. (2023). Eksperymentalne doslidzhennia protsesu pravky prokatu na bahatorolykovykh pravynykh mashynakh [Experimental study of the process of straightening rolled products on multi-roller straightening machines]. *Obrobka materialiv tyskom*, 1(52), 138–144. Retrieved from <https://www.scilit.com/publications/d8c1930b630323d60c993771b9a2cec8>. doi:10.37142/2076-2151/2023-1(52)138 [in Ukrainian].
9. Graça, A., & Vincze, G. (2021). A short review on the finite element method for asymmetric rolling processes. *Metals*, 11(5), 762. <https://doi.org/10.3390/met11050762>.
10. Muraoka, T., Okude, Y., Kajikawa, S., & Kuboki, T. (2023). Effect of initial cross-sectional shape on bent shape in 'Bending and Compression Method' for forming in-plane bent sheet metal. In: *Proceedings of the 14th International Conference on the Technology of Plasticity – Current Trends in the Technology of Plasticity*. Springer, 307–316. doi:10.1007/978-3-031-42093-1_30.
11. Li, L., Xie, H., Liu, T., Huo, M., Li, X., Liu, X., Wang, E., Li, J., Liu, H., Sun, L., & Jiang, Z. (2022). Numerical analysis of the strip crown inheritance in tandem cold rolling by a novel 3D multi-stand FE model. *The International Journal of Advanced Manufacturing Technology*, 120, 3683–3704. <https://doi.org/10.1007/s00170-022-08997-5>.
12. Yan, Z., Pan, S., Tang, Y., & Cao, W. (2024). Theoretical study of asymmetric bending force on metal deformation in cold rolling. *Metals*, 14(10), 1168. <https://doi.org/10.3390/met14101168>.
13. Haribabu, G. N., Yubero, D. C., Maawad, E., Faria, G. A., Staron, P., Schell, N., Ramadhan, R. S., Cabeza, S., Paecklar, A., Pirling, T., Withers, P. J., & Roy, M. J. (2024). Benchmark Sample Design for the Validation of Residual



Stress Measurements by Diffraction: Insights and Practicalities. *Integrating Materials and Manufacturing Innovation*, 13, 955–968. <https://doi.org/10.1007/s40192-024-00385-z>.

14. Silva, R. F., Coelho, P. G., Conde, F. M., Santos, B. R., & Oliveira, J. P. (2024). Minimizing the maximum von Mises stress of elastic continuum structures using topology optimization and additively manufactured functionally graded materials. *Computers & Structures*, 301, 107469. <https://doi.org/10.1016/j.compstruc.2024.107469>.

15. Roubíček, T., & Tomassetti, G. (2023). Inhomogeneous finitely strained thermoplasticity with hardening by an Eulerian approach. *arXiv preprint*, arXiv:2304.05918. <https://doi.org/10.48550/arXiv.2304.05918>.

16. Carvalho, L. A., & Lukács, Z. (2022). The role of friction in the sheet metal forming numerical simulation. *IOP Conference Series: Materials Science and Engineering*, 1246(1), 012021. <https://doi.org/10.1088/1757-899X/1246/1/012021>.

Об'єктом дослідження є процес формоутворення профільних фальцювальних планок шляхом деформування металевих пластин із використанням механіко-математичного моделювання. Метою є встановлення впливу фізико-механічних властивостей матеріалів і типу профілю на характер розподілу напружень та формування геометрії у процесі згинання.

Ключові слова: профільне формоутворення; фальцювальні планки; метод скінченних елементів; моделювання деформації; фізико-механічні властивості.

Надійшла до редакції 07.11.24