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APPLIED COMPUTER POLARIZATION-SINGULAR ANALYSIS OF POLYMER PACKAGING MATERIALS

This manuscript discusses the development of a singular approach to analyzing polarization-inhomogeneous laser fields in order to improve the manufacturing technology of packaging printing products. The analysis is based on a model approach, which represents polyethylene polymer film networks as a two-component amorphous-crystallite matrix.

Keywords: protection technology; graphic information processing; applied programming; printing and packaging materials; materials science.

Introduction

It seems that the article you are referring to discusses the topographic approach to the formation of polarization structures in object fields, particularly in the context of linear and circular polarization singularities of light oscillations. The approach uses S-contours, closed contours in planes where polarization remains linear, to separate sections with right-hand and left-hand circular and elliptical polarization [1–3]. Depending on the scale, shape, distribution, and frequency of repetition of these zones, there can be a deterministic, statistical, or fractal distribution of polarization states in the object field [4–6].

The article also presents a singular approach to analyzing polarization-inhomogeneous laser fields for the purpose of diagnosing linear and circular phase anisotropy of polyethylene polymer film networks. The goal is to improve the manufacturing technology of packaging printing products [7–10].

Method

The Jones operator is a mathematical representation of the effect of an optical element on a polarized light wave. It is a 2×2 matrix that describes the amplitude and phase changes experienced by the electric field components of a polarized light wave as it passes through the element. In the case of optically uniaxial birefringent crystallites, the jones operator depends on the orientation of the optic axis of the crystallite with respect to the polarization direction of the incident light wave [3].

The amorphous component of the matrix is a disordered structure that scatters light, while the crystalline component is an ordered structure that can interact with the polarization of light. The birefringent fibrous networks within the polymer films are responsible for the formation of the polarization-singular structure of the field.

By analyzing the properties of the jones operator at each point in the matrix, it is possible to understand the mechanisms of formation of the polarization structure and to diagnose the linear and circular phase anisotropy of the polymer film networks. This information can be used to improve the manufacturing technology of packaging printing products [7].

$$\{D\} = \left\| \begin{matrix} \cos^2\gamma + \sin^2\gamma \exp(-i\phi); & \cos\gamma \sin\gamma[1-\exp(-i\phi)]; \\ \cos\gamma \sin\gamma[1-\exp(-i\phi)]; & \sin^2\gamma + \cos^2\gamma \exp(-i\phi); \end{matrix} \right\| \quad (1)$$

where γ — the laying direction of birefringent fibers in the plane of the polymer film sample, the substance of which introduces a phase shift between the orthogonal components of the laser beam polarization.

According to (1), the structure of the image of polymer films, including both amplitude and phase information, can be expressed using matrix equation,

$$\begin{pmatrix} E_x(r) \\ E_y(r) \end{pmatrix} = \{C\} \begin{pmatrix} E_{ox} \\ E_{oy} \end{pmatrix}, \quad (2)$$

where E_x, E_y — complex orthogonally polarized components of laser oscillations of the amplitude of the electric intensity vector at the points of the image of polymer films.

To provide a more concise and clear description of the mechanisms of polarization structure formation in polycrystalline fibrous networks of polymer films, we will focus on the case where the film is illuminated by a linearly polarized laser beam. We will assume that the polarization direction of the laser beam is at an angle with respect to the plane of incidence. This assumption does not limit the completeness of our analysis.

$$\begin{pmatrix} E_{ox} \\ E_{oy} \end{pmatrix} \Rightarrow \begin{pmatrix} E_{ox} \\ 0 \end{pmatrix}. \quad (3)$$

Taking into account (3), relation (2) is rewritten in the form

$$\begin{aligned} E_x(r) &= E_{0x}\left[\cos^2\gamma(r) + \sin^2\gamma(r)\exp\left(-i\varphi(r)\right)\right]; \\ E_y(r) &= E_{0y}\left[\cos\gamma(r)\sin\gamma(r)(1-\exp\left(-i\varphi(r)\right))\right]. \end{aligned} \quad (4)$$

In order to determine the local states of polarization of light vibrations at the image points of polymer films, we write the coherence matrices corresponding to them [10]

$$\{K(r)\} = \begin{matrix} E_x(r)E_x^*(r); & E_x(r)E_y^*(r); \\ E_x^*(r)E_x(r); & E_y(r)E_y^*(r); \end{matrix}. \quad (5)$$

According to (5), the coordinate distribution of the azimuth $\alpha(r_i)$ and ellipticity $\beta(r_i)$ of the polarization of the image of polymer films is represented as

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$$\begin{cases} \alpha(r_i) = 0,5\operatorname{arctg}\left[\dfrac{E_x(r_i)E_y^*(r_i) - E_x^*(r_i)E_y(r_i)}{E_x(r_i)E_x^*(r_i) - E_y(r_i)E_y^*(r_i)}\right]; \\ \beta(r_i) = 0,5\arcsin\left[\dfrac{i\left(E_x(r_i)E_y^*(r_i) - E_x^*(r_i)E_y(r_i)\right)}{(q_1 + q_2 + q_3)^{\frac{1}{2}}}\right], \end{cases} \quad (6)$$

where

$$\begin{aligned} q_1 &= \left[E_x(r_i)E_x^*(r_i) - E_y(r_i)E_y^*(r_i)\right]^2; \\ q_2 &= \left[E_x(r_i)E_y^*(r_i) - E_x^*(r_i)E_y(r_i)\right]^2; \\ q_2 &= i\left[E_x(r_i)E_y^*(r_i) - E_x^*(r_i)E_y(r_i)\right]^2. \end{aligned} \quad (7)$$

Here $r_i \equiv \begin{pmatrix} r_{11}, & \dots & r_{1m} \\ \dots & \dots & \dots \\ r_{n1}, & \dots & r_{nm} \end{pmatrix}$ — a set of coordinates determined by the number of pixels of a CCD — a camera that records an image of polymer films.

Taking into account (1)–(7), it can be shown that the conditions for the formation of linear polarization singularities (L-states) are determined by the following relations

$$\sin^2\gamma\cos^2\gamma\left(E_{0x}^2\cos^2\gamma - E_{0y}^2\sin^2\gamma\exp(-2i\varphi_0)\right)\sin^2\varphi = 0. \quad (8)$$

$$\begin{aligned} \gamma &= \operatorname{arctg}\left[\exp(-2i\varphi_0)\frac{E_{0y}}{E_{0x}}\right]; \\ \varphi &= 2q\pi, q = 0, \pm1, 2, \dots \end{aligned} \quad (9)$$

For circular polarization singularities (C-states), the following relation holds

$$\sin^2\gamma\cos^2\gamma\left(E_{0x}^2\cos^2\gamma - E_{0y}^2\sin^2\gamma\exp(-2i\varphi_0)\right)\sin^2\varphi = 1. \quad (10)$$

from which the conditions for the formation of circularly polarized image points of polymer films are determined

$$\begin{aligned} \gamma &= \pi/4 + \operatorname{arctg}\left[\exp(-2i\varphi_0)\frac{E_{0y}}{E_{0x}}\right]; \\ \varphi &= \pi/2 + 2q\pi, q = 0, \pm1, 2, \dots \end{aligned} \quad (11)$$

Our analysis has shown the criteria for the emergence of polarization-singular states networks in polycrystalline networks of polymer films, and we have described the primary mechanisms underlying their topographic construction.

Results

The article presents experimental evidence in the form of examples that demonstrate the existence of characteristic grids of vector-parametric and Mueller-matrix image values for polyethylene polymer films. The examples include films with both ordered fibrous networks that were mechanically stretched (group 1) and disordered fibrous networks that

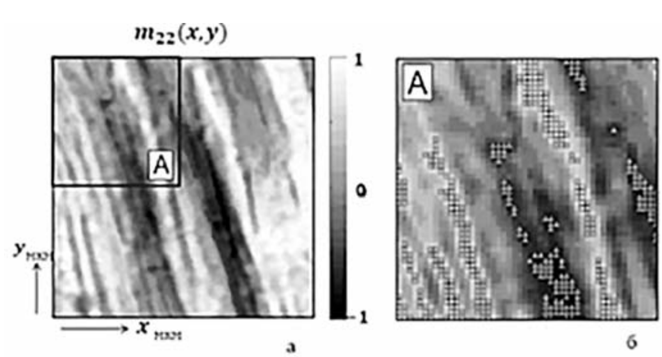

Fig. 1. The coordinate distribution of the element m_{22} of the Mueller matrix of a polyethylene film from group 1, $m_{22} = 1$ denoted by (□); $m_{22} = 0$ denoted as (Δ)

were mechanically undeformed (group 2). On fig. 1–6 are shown the coordinate distributions of the elements of the Mueller matrix of polyethylene films from group 1 (fig. 1, 2) and group 2 (fig. 3–6).

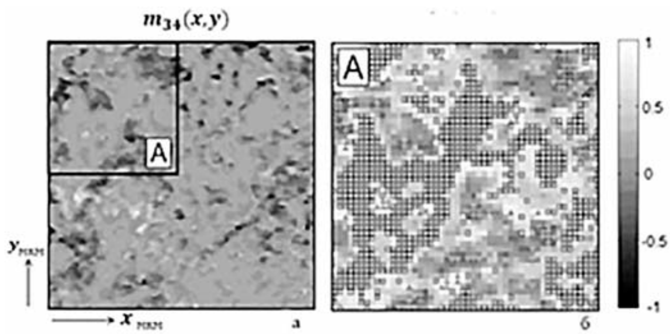

Fig. 4. The coordinate distribution of the element m_{34} of the Mueller matrix of a polyethylene film from group 2, m_{34} = 1 denoted by (Δ); m_{34} = –1 denoted as (□)

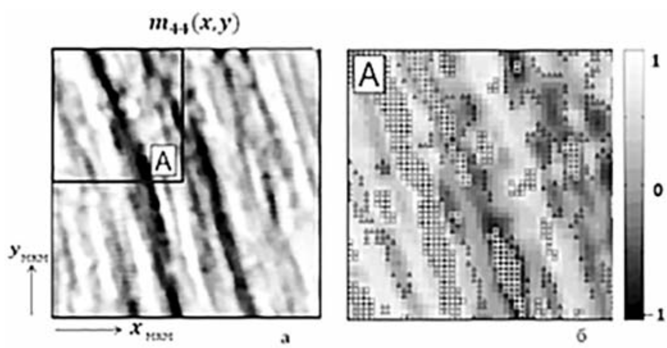

Fig. 2. The coordinate distribution of the element m_{44} of the Mueller matrix of a polyethylene film from group 1, m_{44} = 1 denoted by (□); m_{44} = 0 denoted as (Δ)

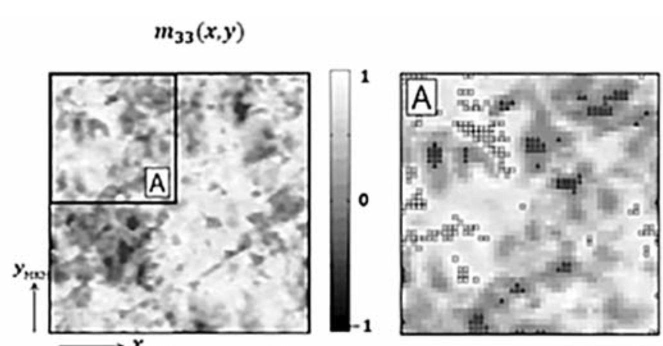

Fig. 3. The coordinate distribution of the element m_{33} of the Mueller matrix of a polyethylene film from group 2, m_{33} = 1 denoted by (□); m_{33} = 0 denoted as (Δ)

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Discussion

An analysis of relations (6)–(11) shows that the main role in the transformation of the laser radiation polarization state is played by the network of birefringent fibers of the polymer film.

Among the various values there are certain special (characteristic) values of the orientational and phase parameters of the polycrystalline fibrous network of the polymer.

$$\begin{cases} \gamma^* = 0°; \pm 45°; 90°; \\ \varphi^* = 0°; \pm 90°; 180°. \end{cases} \quad (12)$$

As can be seen, relation (12) is a necessary and sufficient condition describing the mechanisms of the formation of polarization-singular states at points of a laser image of a polycrystalline network (L – (φ^* = = 0°; 180°) and (±C – (φ^* = ±90°) states).

Such polarization-singular states of the points of the laser image correspond to certain extreme or characteristic values of the fourth parameter of the Stokes vector, associated with the corresponding values of the elements of the Mueller matrix (table 1).

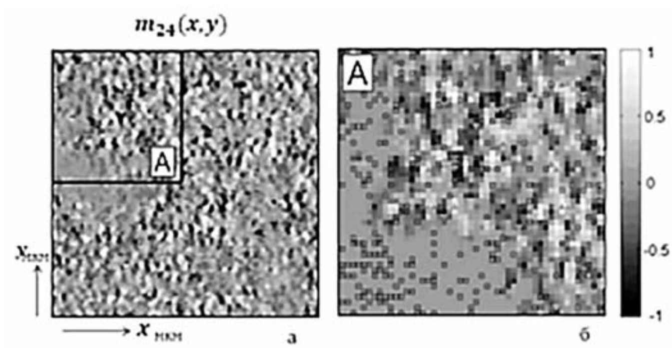

Fig. 5. The coordinate distribution of the element m_{24} of the Mueller matrix of a polyethylene film from group 2, m_{24} = 1 denoted by (Δ); m_{24} = –1 denoted as (∇); m_{24} = 0 denoted as (□)

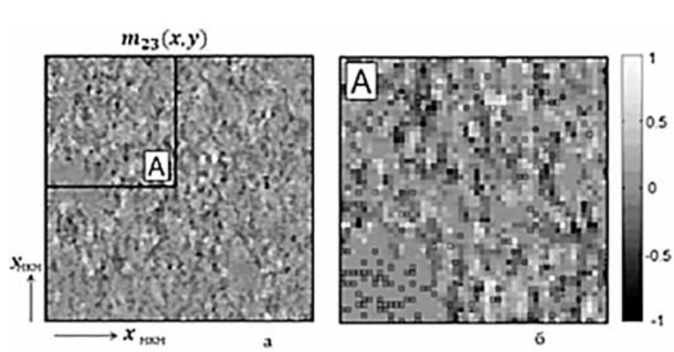

Fig. 6. The coordinate distribution of the element m_{23} of the Mueller matrix of a polyethylene film from group 2, m_{23} = 1 denoted by (Δ); m_{23} = –1 denoted as (∇); m_{23} = 0 denoted as (□)

$$S_4 = \sin 2\beta = \begin{cases} S_4\left(\varphi^* = 0^\circ\right) = 0 \Leftrightarrow L - \text{стан}; \\ S_4\left(\varphi^* = 90^\circ\right) = +1{,}0 \Leftrightarrow +C - \text{стан}; \\ S_4\left(\varphi^* = -90^\circ\right) = -1{,}0 \Leftrightarrow -C - \text{стан}. \end{cases} \quad (13)$$

$$\{M\} = \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & m_{22} & m_{23} & m_{24} \\ 0 & m_{32} & m_{33} & m_{34} \\ 0 & m_{42} & m_{43} & m_{44} \end{matrix} =$$

$$= \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & (\cos^2 2\gamma + \sin^2 2\gamma \cos\varphi); & \cos 2\gamma \sin 2\gamma (1-\cos\varphi); & \sin 2\gamma \sin\varphi \\ 0 & \cos 2\gamma \sin 2\gamma (1-\cos\varphi); & (\sin^2 2\gamma + \cos^2 2\gamma \cos\varphi); & \cos 2\gamma \sin\varphi \\ 0 & -\sin 2\gamma \sin\varphi; & -\cos 2\gamma \sin\varphi; & \cos\varphi \end{matrix}. \quad (14)$$

Table 2 illustrates the characteristic values of Stokes-parametric images of a polycrystalline network.

By establishing the grids of characteristic values for Stokes parameters and Mueller matrices of a polymer polycrystalline network, it becomes feasible to anticipate the formation of polarization singularities in the object field and define

Table 1

Interrelations between the characteristic values of the elements of the Mueller matrix of polyethylene polymer films and the singularity of their polarization-inhomogeneous images

m_{ik}		'L'-states ($\varphi = 0$, $\varphi = \pi$)	'±C'-states $\varphi = \pm \pi/2$
m_{22}	0	—	$\gamma = \pm \pi/4$
	1	$\varphi = 0$	—
	-1	$\varphi = \pi$	—
$m_{24} = -m_{24}$	0	$\gamma = 0, \pi$	—
	1	—	$\gamma = \pm \pi/4$
	-1	—	$\gamma = \mp \pi/4$
m_{44}	0	—	$\gamma = 0 \div \pi$
	1	$\varphi = 0$	—
	-1	$\varphi = \pi$	—

Table 2

Relationship between the anisotropy parameters of a polycrystalline network and the characteristic values of its vector-parametric images

S_i	$\varphi = 0$	$\varphi = \pi/2, \gamma = \pi/4$	$\varphi = \pi/2, \gamma = -\pi/4$
$S_2 = \cos 2\alpha \cos 2\beta$	$\cos 2\alpha$	0	0
$S_3 = \sin 2\alpha \cos 2\beta$	$\sin 2\alpha$	0	0
$S_4 = \sin 2\beta$	0	1	-1

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standards for detecting variations in optical anisotropy parameters.

Conclusions

1. Using a singular approach, the Mueller-matrix analysis of laser radiation's interaction with polycrystalline polymer layers identified the conditions and orientation-phase mechanisms for generating networks of polarization-singular states in corresponding images of such structures.

2. An analytical relationship has been established between polarization-singular states and characteristic values of a set of vector-parametric and Mueller-matrix images of optically anisotropic polymer layers, under the approximation of linear birefringence.

3. Experimental confirmation of the presence of polarization singular states in images of polyethylene polymer layers has been obtained.

4. The correlation between networks of characteristic values of vector-parametric and Mueller-matrix images of a birefringent fibrous network of polyethylene, which is ordered along the directions of optical axes, has been determined.

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У статті описана розробка та обґрунтування принципів сингулярного підходу до аналізу поляризаційно-неод-

норідних лазерних полів з метою вдосконалення технології виготовлення пакувальної поліграфічної продукції. У статті наведено детальні теоретичні пояснення, які допомагають зрозуміти механізми формування поляризаційно-сингулярних структур у полімерних плівках.

Ключові слова: технологія захисту; опрацювання графічної інформації; прикладне програмування; поліграфічні і пакувальні матеріали; матеріалознавство.

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