



UDC 681.7: 535.5

DOI: 10.20535/2077-7264.1(75).2022.425730

© **O. H. Ushenko, Doctor of physics and mathematics of science, Professor, M. P. Horskyi, PhD in physics and mathematics science, Associate professor, I. V. Soltys, PhD in physics and mathematics science, Associate professor, O. V. Dubolazov, Doctor of physics and mathematics of science, Associate professor, Chernivtsi National University, Chernivtsi, Ukraine**

3D POLARIZATION ALGORITHMS FOR PROCESSING DIGITAL MICROSCOPIC IMAGES OF POLYGRAPHIC POLYMERS

A method of azimuthally invariant 3D Mueller-matrix mapping of the distributions of the parameters of phase and amplitude anisotropy of partially depolarizing layers of high-quality (group 1 — high density) and low-quality (group 2 — low density) polyethylene polymer films has been proposed and substantiated. Layer-by-layer coordinate distributions of the magnitude of the set of Mueller-matrix invariants (MMI) of polymer films of both types were obtained in the volume of polymer samples.

Keywords: polarization; 3D Mueller matrix mapping; organic polymers; complex amplitude.

Introduction

At present, optics is actively developing methods and means of polarimetric diagnostics of the structure of polymeric materials, which includes a number of original directions: Mueller-matrix polarimetry [1–5]; two-dimensional Muller-matrix mapping in the framework of various model approximations [6–10].

Our article is aimed at the development and experimental testing of a set of methods of Stokes-polarimetry and interferometry using algorithms for digital holographic reconstruction of the amplitude-phase structure of object fields for differential diagnostics of

layers of high-quality (group 1 — high density) and low-quality (group 2 — low density) of films of polymer polyethylene by obtaining 3D distributions of Mueller-matrix invariants.

Methods

For the Mueller matrix $\{M\}$, the azimuthally invariant, independent of the angle (Θ) of rotation of the sample of the layer, are the following elements M_{ik} and their combinations

$$\{M\} = \begin{pmatrix} 1 & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{pmatrix}. \quad (1)$$



The MMI that characterize the optical anisotropy of organic layers include:

— Matrix elements

$$M_{11}; M_{14}; M_{41}; M_{44}. \quad (2)$$

— Combination of matrix elements

$$\Sigma \equiv (M_{22} + M_{33}); \quad (3)$$

$$\leq \equiv (M_{23} - M_{32}). \quad (4)$$

— Lengths of matrix vectors

$$\begin{cases} A_h = \sqrt{M_{12}^2 + M_{13}^2}; \\ A_v = \sqrt{M_{21}^2 + M_{31}^2}; \\ B_h = \sqrt{M_{42}^2 + M_{43}^2}; \\ B_v = \sqrt{M_{24}^2 + M_{34}^2} \end{cases}. \quad (5)$$

— Angles

$$\begin{aligned} \cos(B_h, B_v) &= \\ &= \frac{-\sqrt{(M_{42}^2 + M_{43}^2)}}{\sqrt{(M_{24}^2 + M_{34}^2)}}. \end{aligned} \quad (6)$$

$$\begin{cases} \{A_h\} = \frac{1}{\sqrt{M_{12}^2 + M_{13}^2}} \begin{pmatrix} M_{12}^2 - M_{13}^2 \\ 2M_{12}M_{13} \end{pmatrix}; \\ \{A_v\} = \frac{1}{\sqrt{M_{21}^2 + M_{31}^2}} \begin{pmatrix} M_{21}^2 - M_{31}^2 \\ 2M_{21}M_{31} \end{pmatrix}; \\ \{B_h\} = \frac{1}{\sqrt{M_{42}^2 + M_{43}^2}} \begin{pmatrix} M_{42}^2 - M_{43}^2 \\ 2M_{42}M_{43} \end{pmatrix}; \\ \{B_v\} = \frac{1}{\sqrt{M_{24}^2 + M_{34}^2}} \begin{pmatrix} M_{24}^2 - M_{34}^2 \\ 2M_{24}M_{34} \end{pmatrix} \end{cases}. \quad (7)$$

$$G = \sqrt{(M_{22} - M_{33})^2 + (M_{23} - M_{32})^2}. \quad (8)$$

The use of the MMI set will provide conditions for the dissemination of methods of experimentally reproducible Mueller-matrix mapping to serial, screening investigations.

It is based on the use of a reference wave of laser radiation, which in the scheme of an optical interferometer is superimposed on a polarization-inhomogeneous image of a polymer film. The resulting interference pattern is recorded using a digital camera. With the help of diffraction integrals, the digital holographic reproduction of the distributions of the complex amplitudes $\{E_x(x, y); E_y(x, y)\}$ of the object field of the polymer layer takes place.

The set of elements of the Muller matrix is calculated by the following Stokes-polarimetric relations:

— for the Stokes vectors of linearly polarized probing beams $s^0(0^\circ); s^0(90^\circ)$

$$\begin{aligned} \left[\begin{array}{l} S^0(0^\circ) = \{M\} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow S(0^\circ) = \begin{pmatrix} M_{11} + M_{12} \\ M_{21} + M_{22} \\ M_{31} + M_{32} \\ M_{41} + M_{42} \end{pmatrix} \\ S^0(90^\circ) = \{M\} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \rightarrow S(90^\circ) = \begin{pmatrix} M_{11} - M_{12} \\ M_{21} - M_{22} \\ M_{31} - M_{32} \\ M_{41} - M_{42} \end{pmatrix} \end{array} \right] \Rightarrow \\ \Rightarrow M_k = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \\ M_{31} & M_{32} \\ M_{41} & M_{42} \end{pmatrix}, \end{aligned} \quad (9)$$

— for the Stokes vectors of linearly polarized probing beams $s^0(45^\circ); s^0(135^\circ)$:

$$\begin{aligned} \left[\begin{array}{l} S^0(45^\circ) = \{M\} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow S(45^\circ) = \begin{pmatrix} M_{11} + M_{13} \\ M_{21} + M_{23} \\ M_{31} + M_{33} \\ M_{41} + M_{43} \end{pmatrix} \\ S^0(135^\circ) = \{M\} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \rightarrow S(135^\circ) = \begin{pmatrix} M_{11} - M_{13} \\ M_{21} - M_{23} \\ M_{31} - M_{33} \\ M_{41} - M_{43} \end{pmatrix} \end{array} \right] \Rightarrow \\ \Rightarrow M_k = \begin{pmatrix} M_{11} & M_{13} \\ M_{21} & M_{23} \\ M_{31} & M_{33} \\ M_{41} & M_{43} \end{pmatrix}, \end{aligned} \quad (10)$$

— for the Stokes vectors of right and left circularly polarized probing beams $S^0(\otimes); S^0(\oplus)$:



$$\left[\begin{array}{l} S^{\circ}(\otimes) = \{M\} \\ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \end{array} \right] \rightarrow S(\otimes) = \begin{pmatrix} M_{11} + M_{14} \\ M_{21} + M_{24} \\ M_{31} + M_{34} \\ M_{41} + M_{44} \end{pmatrix};$$

$$\left[\begin{array}{l} S^{\circ}(\oplus) = \{M\} \\ \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \end{array} \right] \rightarrow S(\oplus) = \begin{pmatrix} M_{11} - M_{14} \\ M_{21} - M_{24} \\ M_{31} - M_{34} \\ M_{41} - M_{44} \end{pmatrix};$$

$$\Rightarrow M_k = \begin{pmatrix} M_{11} & M_{14} \\ M_{21} & M_{24} \\ M_{31} & M_{34} \\ M_{41} & M_{44} \end{pmatrix}. \quad (11)$$

For an objective assessment of layer-by-layer polarization maps $S(\theta_k, x, y)$, the statistical moments of the first (z_1), second (z_2), third (z_3) and fourth (z_4) orders were used, which were calculated by the following algorithms [8]

$$Z_1 = \frac{1}{N} \sum_{j=1}^N S(\theta_k, x, y)_j;$$

$$Z_2 = \sqrt{\frac{1}{N} \sum_{j=1}^N (S^2(\theta_k, x, y))_j};$$

$$Z_3 = \frac{1}{Z_2} \frac{1}{N} \sum_{j=1}^N (S^3(\theta_k, x, y))_j;$$

$$Z_4 = \frac{1}{Z_2} \frac{1}{N} \sum_{j=1}^N (S^4(\theta_k, x, y))_j, \quad (12)$$

where N — number of pixels of the photosensitive area of the CCD camera.

Results

In order to determine the diagnostic efficiency of the 3D Mueller-matrix mapping method in differentiating layers of high-quality (group 1 — high density) and low-quality (group 2 — low density) of polyethylene polymer films, two groups of partially depolarizing (degree of depolarization $\Lambda \leq 50\%$) layers were formed:

— 26 samples — group 1 (attenuation coefficient $0,79 < \tau < 0,85$, $43\% < \Lambda < 48\%$);

— 26 samples — group 2 ($0,81 < \tau < 0,84$, $45\% < \Lambda < 47\%$).

Optical technology for differential diagnosis of such samples includes the following steps:

1. Determination of a series of «phase» layer-by-layer images of 3D MMI distributions $\{M_{44}; \Delta M; M_{41}; M_{14}\}$ ($\varphi_1 = 0,3; 2h\varphi_1, \dots, 6\varphi_1$) characterizing volumetric polarization manifestations of phase and amplitude anisotropy within both groups of samples.

2. For each «phase» section of 3D distributions of the MMI value, a set of statistical moments of the 1st–4th orders is calculated $Z_{i=1;2;3;4}\{[M_{44}; \Delta M; M_{41}; M_{14}](\varphi_k, x, y)\}$.

3. For samples of group 1 and group 2, «phase» dependences $Z_{i=1;2;3;4}\{[M_{44}; \Delta M; M_{41}; M_{14}](\varphi_1, \varphi_2, \dots, \varphi_k)\}$ of the magnitude of each statistical moment are plotted.

4. The «phase» planes (φ^*) are determined in 3D MMI distributions, where the maximum differences between the values of the statistical moments ($\Delta Z_{i=1;2;3;4}^* \equiv \Delta Z_{i=1;2;3;4}(\varphi^*) \rightarrow \max$), which characterize the distributions of the values of matrix elements $M_{44}; \Delta M; M_{41}; M_{14}$ in these planes, are realized.

5. In the «phase» plane φ^* , the average $\Delta Z_{i=1;2;3;4}^*$ and the error

$\sigma(\Delta Z_i^*)$ are determined within the polymer films from group 1 and group 2.

Discusions

The «phase» dependences of the magnitude of the statistical moments of the 1st–4th orders,



Statistical criteria for the differentiation of polymer films based on phase anisotropy

Parameters	Group 1		Group 2		Accuracy, Ac, %	
	M ₄₄	ΔM	M ₄₄	ΔM	M ₄₄	ΔM
Z ₁ (φ* = 0,45)	0,29±0,017	0,12±0,007	0,44±0,029	0,07±0,004	85	82
Z ₂ (φ* = 0,45)	0,21±0,012	0,15±0,008	0,14±0,006	0,11±0,005	81	79
Z ₃ (φ* = 0,45)	0,46±0,029	0,63±0,041	0,69±0,037	0,92±0,055	91	89
Z ₄ (φ* = 0,45)	0,57±0,033	0,88±0,053	1,03±0,059	1,39±0,084	92	87

characterizing the distributions of the MMI values of the polarization manifestations of the parameters of the linear and circular birefringence and dichroism of the polycrystalline component of various types of polyethylene layers have been determined.

The optimal conditions for the differentiation of polycrystalline structures of polymer layers of different densities — the range of phase cross-sections and the most sensitive parameters — statistical moments of the 3rd and 4th orders, characterizing the distributions of MMI are revealed.

References

1. Manhas, S. & others (2006). Mueller matrix approach for determination of optical rotation in chiral turbid media in backscattering geometry. *Opt. Express*, 14(1), 190–202.
2. Deng, Y., & others (2007). Characterization of backscattering Mueller matrix patterns of highly scattering media with triple scattering assumption. *Opt. Express*, 15(15), 9672–9680.
3. Ushenko, A. G., & Pishak, V. P. (2004). *Laser Polarimetry of Biological Tissue: Principles and Application*. Handbook of Coherent-Domain Optical Methods: Biomedical Diagnostics, Environmental and Material Science. Boston: Kluwer Academic Publishers, vol. I, 93–138.
4. Angelsky, O. V., Ushenko, A. G., Ushenko, Yu. A., Pishak, V. P., & Peresunko, A. P. (2010). *Statistical, Correlation and Topological Approaches in*

Conclusion

1. A method of azimuthally invariant 3D Mueller matrix mapping of the distributions of the parameters of phase and amplitude anisotropy of partially depolarizing layers of qualitative (group 1 — high density) and low-quality (group 2 — low density) polyethylene polymer films is proposed and substantiated.

2. Layer-by-layer coordinate distributions of the set of Mueller-matrix invariants of polyethylene were obtained in the volume of film samples.



Diagnostics of the Structure and Physiological State of Birefringent Biological Tissues. Handbook of Photonics for Biomedical Science. Boca Raton, London, New York: CRC PressTaylor&Francis group, 283–322.

5. Ushenko, Y. A., Boychuk, T. M., Bachynsky, V. T., & Mincer, O. P. (2013). *Diagnostics of Structure and Physiological State of Birefringent Biological Tissues: Statistical, Correlation and Topological Approaches*. Handbook of Coherent-Domain Optical Methods. New York: Springer Science + Business Media, 107.

6. Lu, S. Y., & Chipman, R. A. (1996). Interpretation of Mueller matrices based on polar decomposition. *J. Opt. Soc. Am. A*, 13(5), 1106–1113.

7. Guo, Y., & others (2013). A study on forward scattering Mueller matrix decomposition in anisotropic medium. *Opt. Express*, 21(15), 18361–18370.

8. Deboo, B., Sasian, J., & Chipman, R. A. (2004). Degree of polarization surfaces and maps for analysis of depolarization. *Opt. Express*, 12(20), 4941–4958.

9. Buscemi, I. C., & Guyot, S. (2013). Near real-time polarimetric imaging system. *J. Biomed. Opt.*, 18(11), 116002.

10. Prysyzhnyuk, V. P., Ushenko, Yu. A., Dubolazov, A. V., Ushenko, A. G., & Ushenko, V. A. (2016). Polarization-dependent laser autofluorescence of the polycrystalline networks of blood plasma films in the task of liver pathology differentiation. *Appl. Opt.*, 55, B126-B132.

У статті представлено матеріали аналітичного обґрунтування та експериментальної апробації нового поляриметричного методу азимутально-інваріантного 3D-матричного картування Мюллера розподілів параметрів фазової та амплітудної анізотропії частково деполаризуючих шарів високоякісних (1 група — висока щільність) і низькоякісних плівок.

Ключові слова: поляризація; 3D матричне відображення Мюллера; органічні полімери; комплексна амплітуда.

Надійшла до редакції 07.05.22