Introduction

At present, optics is actively developing methods and means of polarimetric diagnostics of the structure of polymeric materials, which includes a number of original directions: Mueller-matrix polarimetry [1–5]; two-dimensional Muller-matrix mapping in the framework of various model approximations [6–10].

Our article is aimed at the development and experimental testing of a set of methods of Stokes-polarimetry and interferometry using algorithms for digital holographic reconstruction of the amplitude-phase structure of object fields for differential diagnostics of layers of high-quality (group 1 — high density) and low-quality (group 2 — low density) polyethylene polymer films by obtaining 3D distributions of Mueller-matrix invariants.

Methods

A method of azimuthally invariant 3D Mueller-matrix mapping of the distributions of the parameters of phase and amplitude anisotropy of partially depolarizing layers of high-quality (group 1 — high density) and low-quality (group 2 — low density) polyethylene polymer films has been proposed and substantiated. Layer-by-layer coordinate distributions of the magnitude of the set of Mueller-matrix invariants (MMI) of polymer films of both types were obtained in the volume of polymer samples.

Keywords: polarization; 3D Mueller matrix mapping; organic polymers; complex amplitude.
The MMI that characterize the optical anisotropy of organic layers include:
- Matrix elements
  \[ M_{11}; M_{14}; M_{41}; M_{44}. \]  
- Combination of matrix elements
  \[ \Sigma \equiv (M_{22} + M_{33}); \quad \kappa \equiv (M_{23} - M_{32}). \]  
- Lengths of matrix vectors
  \[
  \begin{align*}
  A_h &= \sqrt{M_{11}^2 + M_{22}^2}, \\
  A_v &= \sqrt{M_{33}^2 + M_{44}^2}, \\
  B_h &= \sqrt{M_{22}^2 + M_{33}^2}, \\
  B_v &= \sqrt{M_{44}^2 + M_{22}^2}.
  \end{align*}
  \]  
- Angles
  \[
  \cos(B_h, B_v) = \frac{-\sqrt{(M_{22}^2 + M_{44}^2)}}{\sqrt{(M_{22}^2 + M_{44}^2)}}.
  \]

The use of the MMI set will provide conditions for the dissemination of methods of experimentally reproducible Mueller-matrix mapping to serial, screening investigations. It is based on the use of a reference wave of laser radiation, which in the scheme of an optical interferometer is superimposed on a polarization-inhomogeneous image of a polymer film. The resulting interference pattern is recorded using a digital camera. With the help of diffraction integrals, the digital holographic reproduction of the distributions of the complex amplitudes \( \{E_x(x, y); E_y(x, y)\} \) of the object field of the polymer layer takes place.

The set of elements of the Muller matrix is calculated by the following Stokes-polarimetric relations:
- for the Stokes vectors of linearly polarized probing beams \( s^0(0^0); s^0(90^0) \):
  \[
  \begin{align*}
  &s^0(0^0) - (M) \rightarrow \mathbf{S}^0(0^0) - \begin{pmatrix} M_{11} + M_{22} \\ M_{11} - M_{22} \\ M_{33} + M_{44} \\ M_{33} - M_{44} \end{pmatrix} \\
  &s^0(90^0) - (M) \rightarrow \mathbf{S}^0(90^0) - \begin{pmatrix} M_{11} + M_{22} \\ M_{11} - M_{22} \\ M_{33} + M_{44} \\ M_{33} - M_{44} \end{pmatrix} \\
  \Rightarrow M_s = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \\ M_{23} & M_{24} \\ M_{34} & M_{44} \end{pmatrix}
  \end{align*}
  \]  
- for the Stokes vectors of right and left circularly polarized probing beams \( s^0(45^0); s^0(135^0) \):
  \[
  \begin{align*}
  &s^0(45^0) - (M) \rightarrow \mathbf{S}^0(45^0) - \begin{pmatrix} M_{11} + M_{12} \\ M_{11} - M_{12} \\ M_{33} + M_{43} \\ M_{33} - M_{43} \end{pmatrix} \\
  &s^0(135^0) - (M) \rightarrow \mathbf{S}^0(135^0) - \begin{pmatrix} M_{11} + M_{12} \\ M_{11} - M_{12} \\ M_{33} + M_{43} \\ M_{33} - M_{43} \end{pmatrix} \\
  \Rightarrow M_s = \begin{pmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \\ M_{23} & M_{24} \\ M_{34} & M_{44} \end{pmatrix}
  \end{align*}
  \]  
- for the Stokes vectors of right and left circularly polarized probing beams \( S^0(\otimes); S^0(\oplus) \):
For an objective assessment of layer-by-layer polarization maps $S(\theta_k, x, y)$, the statistical moments of the first ($z_1$), second ($z_2$), third ($z_3$) and fourth ($z_4$) orders were used, which were calculated by the following algorithms [8]

$$Z_i = \frac{1}{N} \sum_{j=1}^{N} S(\theta_k, x, y);$$

$$Z_2 = \frac{1}{Z_2^2} \sum_{j=1}^{N} (S^2(\theta_k, x, y));$$

$$Z_3 = \frac{1}{Z_3^2} \sum_{j=1}^{N} (S^3(\theta_k, x, y));$$

$$Z_4 = \frac{1}{Z_4^2} \sum_{j=1}^{N} (S^4(\theta_k, x, y));$$

(12)

where $N$ — number of pixels of the photosensitive area of the CCD camera.

**Results**

In order to determine the diagnostic efficiency of the 3D Mueller-matrix mapping layers of high-quality (group 1 — high density) and low-quality (group 2 — low density) of polyethylene polymer films, two groups of partially depolarizing (degree of depolarization $\Lambda \leq 50\%$) layers were formed:

— 26 samples — group 1 ($0.79 < \tau < 0.85$, $43\% < \Lambda < 48\%$);

— 26 samples — group 2 ($0.81 < \tau < 0.84$, $45\% < \Lambda < 47\%$).

Optical technology for differential diagnosis of such samples includes the following steps:

1. Determination of a series of «phase» layer-by-layer images of 3D MMI distributions $\{M_{44}; \Delta M; M_{41}; M_{14}\} (\varphi_1 = 0.3; 2\varphi_1, ..., 6\varphi_1)$ characterizing volumetric polarization manifestations of phase and amplitude anisotropy within both groups of samples.

2. For each «phase» section of 3D distributions of the MMI value, a set of statistical moments of the 1st–4th orders is calculated $Z_{i=1;2;3;4}\{[M_{44}; \Delta M; M_{41}; M_{14}\]}(\varphi_k, x, y)$.

3. For samples of group 1 and group 2, «phase» dependences $Z_{i=1;2;3;4}\{[M_{44}; \Delta M; M_{41}; M_{14}\]}(\varphi_1, \varphi_2, ..., \varphi_k)$ of the magnitude of each statistical moment are plotted.

4. The «phase» planes ($\varphi^*$) are determined in 3D MMI distributions, where the maximum differences between the values of the statistical moments ($\Delta Z_{i=1;2;3;4}(\varphi^*)_{\text{max}}$), which characterize the distributions of the values of matrix elements $M_{44}; \Delta M; M_{41}; M_{14}$ in these planes, are realized.

5. In the «phase» plane $\varphi^*$, the average $\Delta Z_{i=1;2;3;4}$ and the error $\sigma(\Delta Z_{i})$ are determined within the polymer films from group 1 and group 2.

**Discussions**

The «phase» dependences of the magnitude of the statistical moments of the 1st–4th orders,
characterizing the distributions of the MMI values of the polarization manifestations of the parameters of the linear and circular birefringence and dichroism of the polycrystalline component of various types of polyethylene layers have been determined.

The optimal conditions for the differentiation of polycrystalline structures of polymer layers of different densities — the range of phase cross-sections and the most sensitive parameters — statistical moments of the 3rd and 4th orders, characterizing the distributions of MMI are revealed.

**Conclusion**

1. A method of azimuthally invariant 3D Mueller matrix mapping of the distributions of the parameters of phase and amplitude anisotropy of partially depolarizing layers of qualitative (group 1 — high density) and low-quality (group 2 — low density) polyethylene polymer films is proposed and substantiated.

2. Layer-by-layer coordinate distributions of the set of Mueller-matrix invariants of polyethylene were obtained in the volume of film samples.

**References**


У статті представлено матеріали аналітичного обґрунтування та експериментальної апробації нового поляриметричного методу азимутально-інваріантного 3D-матричного картування Мюллера розподілів параметрів фазової та амплітудної анізотропії частково деполяризуючих шарів високоякісних і низькоякісних плівок.

Ключові слова: поляризація; 3D матричне відображення Мюллера; органічні полімери; комплексна амплітуда.

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